WHS-542
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11 Sheets

28 November 1967

Notes on Selection of Targets for Viewing

References: 1. WHS-480, "The Effect of Alternate Targets upon Primary Path Selection," dtd 9 October 67.

2. F. Perkins "Summary of Effective Worth of Primary," dtd 25 October 67.

I. Problem Statement

The requirement has been established that after taking a primary or one of its alternates, as the case may be, the main optics must then be able to acquire the next primary. This requirement geometrically imposes a latest possible decision time upon each alternate. In other words, for any alternate under consideration the decision to take that alternate must be made on or before a certain time or the main optics will not be able to acquire the next primary. This constitutes one limitation. If there are relatively few alternates for each primary, this may be the only limitation that need be considered. In that case multiplexing or other techniques may be used to define various sets of alternates which may be viewed by the ATS up to certain key decision times. The equations of Reference 1 and 2 may then be used to determine the average score for each of these sets and the maximum case selected therefrom.

On the other hand, if there are many alternate targets, the constraint of having to be able to acquire the next primary with the main optics, may not sufficiently limit the number of alternates for any specific decision time. There may be more alternates still available for viewing through the ATS than time permits. The question of which of these alternates should be selected for viewing may not be answered by taking all possible combinations (multiplexing) because of excessive computation time.

The operational flight problem may be stated as follows. Given a target complex of M total targets consisting of one primary and a group of possible alternates. Only P (generally $P < M$) targets may be viewed by the man and each observed to be either active or inactive in his opinion or cloud covered. One and only one target will then be selected to be taken on the basis of greatest probable worth.

The problem at hand is to determine before flight which P targets should be selected for viewing on the basis of greatest average expected score. Reference 1 and 2 present the exact closed form solution for the average expected score once it has been decided which P targets are to be viewed. Manipulation of this solution is used herein to solve the problem of deciding which targets should be selected for viewing.

II. Method of Solution

The solution of Reference 1 and 2 for average effective worth of a complex required that the component worth entries of observed active, observed inactive, or unobserved targets be entered in a single list in a fixed order or hierarchy. That identical hierarchy will be preserved throughout this solution.

The equation for average total score shows clearly that, no matter which targets are viewed, the contribution of any entry is greatly influenced by all entries ahead of it on the final list and that it in turn effects the contributions to average expected score of all entries below it. Therefore, it is presumed that there is no simple closed form equation to tell which targets should be selected for viewing. Basically, the procedure used herein will be to first compose a trial list of P targets with entries nearest the head of the master list. The next target from the master list is now added as a trial value. Successively from top to bottom the effect on total score of deleting each single target including the one just added is determined and the one the deletion of which results in the highest total score is permanently discarded. Then the next target from the master list is added and the procedure repeated until the master list is exhausted. Depending upon the magnitude of the numbers involved, a second pass through the target list may be required. The targets then remaining on the partial list are the ones to be viewed.

Actually the implementation of this procedure is more complex than the foregoing explanation may indicate, because continuously added as the lowest entry on each trial partial list must be the greatest unobserved effective worth selected from the remaining targets on the master list. Also some targets will have two instead of one entry on the trial list as it is being developed. Further, these two entries may have between them the entries of other targets.

The general outline of the following parts of this report is to first present the solution of Reference 1 and 2, then analyze the effect of each target upon the possible value of each other target in the process of selecting targets to be observed and, finally, outline a possible procedure for solution.

Symbols

The following symbols are the same as those of Reference 1 and 2, though the contractor has used other symbols when he programmed this solution for the average expected gain.

W_A	Basic Active Worth
W_I	Basic Inactive Worth
P_A	Probability of Activity
P_V	Probability of Visibility
P_R	Probability of Recognizing Activity
P_F	Probability of False Alarm
ρ_A	Probability Man will Designate Target as Active
ρ_I	Probability Man will Designate Target as Inactive

$\overline{\omega}_A$ — Effective Active Worth of Target which has been Designated as Active

$\overline{\omega}_I$ — Effective Inactive Worth of Target which has been Designated as Inactive

$\omega_A = \overline{\omega}_A \rho_A =$ The part of the Effective Worth of a Target, which is due to the possibility that it might be Designated as Active

$\omega_I = \overline{\omega}_I \rho_I =$ The Part of the Effective Worth of a Target, which is due to the possibility that it might be Designated as Inactive

$\omega = \omega_A + \omega_I = \left[\rho_A W_A + (1-\rho_A) W_I\right]\rho_V =$ Effective Worth of a Target which the man will not attempt to observe.

EFF W_P = Effective Average Worth of Complex or Primary

IIA. Computation of Average Worth of a Defined Group

The Appendix attached hereto is the solution of Reference 2 for the effective average worth of a group of targets when the selection of targets for viewing from within that group has been pre-ordained.

The final equation for this solution may be written in abbreviated notation as follows. Both the Table of Symbols and the Appendix must be read to obtain the meaning of the symbols.

$$\text{EFF } W_P = \omega_{j=1} + \sum_{j=2}^{N} \prod_{i=1}^{j-1} \left(1-\overline{\rho}_i\right) \omega_j$$

This equation may be written in expanded form as follows:

$$
\begin{aligned}
\text{EFF } W_P = \; & \omega_1 \\
& + \left(1-\overline{\rho_1}\right) \omega_2 \\
& + \left(1-\overline{\rho_1}\right)\left(1-\overline{\rho_2}\right) \omega_3 \\
& + \left(1-\overline{\rho_1}\right)\left(1-\overline{\rho_2}\right)\left(1-\overline{\rho_3}\right) \omega_4 \\
& + \left(1-\overline{\rho_1}\right)\left(1-\overline{\rho_2}\right)\left(1-\overline{\rho_3}\right)\left(1-\overline{\rho_4}\right) \omega_5 \\
& + \text{etc.}
\end{aligned}
$$

Each line on the right hand side of the equation represents an entry from the list of ω's as explained in the Appendix. Some targets may have two such entries, not necessarily consecutively. The value of the entry represented by the third line in the above equation has been degraded by the factor,

$$\left(1-\overline{\rho_1}\right)\left(1-\overline{\rho_2}\right) < 1.0$$

$$\text{Always } 0 < \overline{\rho} < 1.0$$

The amount of this degradation depends upon which targets are selected for entries one and two. All entries after the third are degraded by the factor,

$$\left(1-\overline{\rho_3}\right)$$

Thus, the net contribution to total average worth of adding the k'th entry may be precomputed as,

$$\text{NET } W_k = \Delta W_k - \overline{\rho_k} \sum_{i=k+1}^{N} \Delta W_i$$

$$\text{where } \Delta W_i \triangleq \omega_i \prod_{j=1}^{j=i-1} \left(1-\overline{\rho_j}\right)$$

As shown by this equation for NET W_k, adding another entry into the list, diminishes the net contribution of every other entry beneath the one being added and generally lowers the contribution of entries preceding it. Profitable additions to the list may render negative the net contribution of some other target, in which case the latter should be removed from the list.

III. Computational Procedure

1. For each target there will be given a

$$W_A,\ W_I,\ P_A,\ P_V,\ P_R,\ P_F$$

2. Compute for each target,

$$\rho_A = \left[P_A P_R + \left(1 - P_A\right) P_F\right] P_V$$

$$\rho_I = P_V - P_A$$

$$\omega_A = \left[P_A P_R W_A + \left(1 - P_A\right) P_F W_I\right] P_V$$

$$\omega_I = \left[P_A\left(1 - P_R\right) W_A + \left(1 - P_A\right)\left(1 - P_F\right) W_I\right] P_V$$

$$\omega = \left[P_A W_A + \left(1 - P_A\right) W_I\right] P_V$$

$$\overline{\omega}_A = \frac{\omega_A}{\rho_A}$$

$$\overline{\omega}_I = \frac{\omega_I}{\rho_I}$$

3. List all ω, $\overline{\omega}_A$, and $\overline{\omega}_I$, in order of descending magnitude without regard to subscript or superscript. Without changing the order, replace all $\overline{\omega}_A$ and $\overline{\omega}_I$ in this list by the ω_A and ω_I, respectively, for the corresponding targets. This is the master list containing three entries, ω_A, ω_I, and ω, for each target. Throughout the solution various entries will be extracted from this master list and transferred to an ever changing shorter (partial) list in which the same relative order of entries will be maintained.

Generally, the three entries for any target will fall in the order ω_A, ω, ω_I, although it is possible that in some cases the reverse order will result. The relative magnitude of these three entries will not necessarily follow any particular order. Of course, between these three entries may fall some of the entries for other targets.

All three entries for each target must be subscripted or superscripted in some way to indicate to which target they apply. Those with the A or I subscript are referred to as "observed" values, or entries, and those without it as "unobserved". The second observed entry for each target is flagged as a second entry. Whenever an observed entry for a target is transferred from the master list to the trial list or vice versa, the other observed entry for that same target is also transferred. The terminology "next observed target on the list" means the target with an observed entry nearest the top of the list.

The following steps constitute a possible solution:

1. Transfer the P first-entry observed values from the head of the master list to a trial target list and also transfer the second entry observed values for these same targets. In other words, transfer the first P observed targets from the master list to the trial list.

2. Transfer to the trial list the two entries for the next observed target on the master list.

3. Temporarily remove from the trial list the observed target at the head of the list. Transfer from the master list to the trial list the greatest unobserved entry selected from among those targets not represented on the trial list and label it as the Nth entry. Compute the score of the complex represented on the trial list by means of the equation presented in the Appendix. Notice that the computation stops with the Nth entry.

Return the Nth entry to the master list and restore to the trial list the target that was temporarily removed.

4. Temporarily remove from the trial list the second observed target and repeat the process as outlined in Step 3. Continue to repeat this process until a score has been obtained for the removal of each individual target from the trial list. Permanently remove that target, the removal of which results in the highest score.

5. Repeat the entire process starting with Step 2 and continue to repeat it until all of the observed targets on the master list have been transferred to the trial list. The solution is now probably complete and the trial list should be a near optimum selection. If, however, the relative magnitudes of the numbers involved do not render an adequate solution, a single iteration should suffice.

To select P targets from a total of M by the foregoing method requires that the average effective worth (score) be computed (P+1) (M-P) times. This may be reduced slightly by rearranging the procedure, though for computer usage it is probably not worth the bother.

In the foregoing procedure, total average expected score was computed periodically. It is possible to compute only incremental scores caused by the various additions and deletions of entries to the trial list. However, this may result in extra complication if more than one entry per target is actually involved in the computation. If the magnitudes are such that only a single observed value for each target enters into the computation, then incremental scores are sufficient.

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Appendix

Assumptions

1. Certain targets in a group consisting of a primary and several alternates will be pre-selected for viewing by the Man. This analysis does not determine how to select those targets which should be viewed. It determines an average score for the group once the selection has been made. (A crude rule of thumb for selecting which alternates should be viewed if there is not time to view all of them, might be to take those with the higher ω_A.)

2. Operationally only one target will ultimately be taken.

Computation

1. For each target there will be a

$$W_A,\ W_I,\ P_A,\ P_V,\ P_R,\ P_F \text{ (see Table of Symbols)}$$

2. Compute for each target selected for viewing,

$$\rho_A = \left[P_A P_R + (1-P_A) P_F \right] P_V$$

$$\rho_I = P_V - \rho_A$$

$$\omega_A = \left[P_A P_R W_A + (1-P_A) P_F W_I \right] P_V$$

$$\omega_I = \left[P_A (1-P_R) W_A + (1-P_A)(1-P_F) W_I \right] P_V$$

$$\overline{\omega}_A = \frac{\omega_A}{\rho_A}$$

$$\overline{\omega}_I = \frac{\omega_I}{\rho_I}$$

3. Compute for each target not selected for viewing

$$\omega = \left[P_A W_A + (1-P_A) W_I \right] P_V$$

4. From among the targets <u>not</u> selected for viewing, choose the <u>one</u> with the highest ω and designate this single value as $\overline{\omega}$ without a subscript.

5. List all $\overline{\omega}$'s in order of <u>descending</u> magnitude without regard to subscripts A, I, or the one value with no subscript.

6. Remove all entries <u>below</u> the single non-subscripted $\overline{\omega}$.

7. Some targets will have two entries, both an $\overline{\omega}_A$ and an $\overline{\omega}_I$ remaining in this reduced list. For these targets <u>flag</u> the second (lower) entry for later reference.

8. The effective average worth of the complex is as follows. Subscripts A or I are not shown in this equation because they vary.

$$\text{EFF } W_P = \overbrace{\overline{\omega}_{j=1}}^{\omega_{j=1}} \rho_{j=1} + \sum_{j=2}^{N} \prod_{i=1}^{j-1} (1-\overline{\rho}_i) \overbrace{\overline{\omega}_j}^{\omega_j} \rho_j$$

where: i or j is the number of the entry in order of descending mixed $\overline{\omega}_A$, $\overline{\omega}_I$, $\overline{\omega}$

N is the total number of entries

$\overline{\omega}_j$ is either $\overline{\omega}_A$, $\overline{\omega}_I$, or $\overline{\omega}$ whichever applies to the jth entry

ρ_j has the same subscript as $\overline{\omega}_j$. ρ_j is unity for the final entry.

$\overline{\rho}_i$ is ρ_{Ai} or ρ_{Ii} whichever has the correct subscript for the ith entry, <u>except when</u> the repetition flag is on in which case $\overline{\rho}$ is P_V.

Theory

See memo of 9 October 1967 for more detail. The reason the $\overline{\omega}$'s are ordered according to descending magnitude without regard to subscript is that in the operational case the target actually taken will be the first one on this list which the man finds is both visible and has an activity in agreement with its subscript A or I. If none of those viewed by the man qualify, then the final entry (the non-subscripted one) would definitely be taken because it has a higher effective worth than the lower items previously deleted from the list. The probability that this last item is invisible is included in its effective worth.

In the final equation for EFF W_P the terms $(\overline{\omega}_j \rho_j)$ occur. This product is the algebraically simpler term ω_j as indicated.